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AN WIND-INDUCED WAVE SUBMODEL COUPLED TO EFDC

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Abstract

The paper presents a sub-model for wind-induced wave incorporated into EFDC. The module is developed to effectively solve numerous problems in fluid dynamics related to the simulation and prediction of the flow currents and waves in estuaries, rivers, lakes and coastal zones, especially with sediment transport and bed morphology. The model has been calibrated through the comparison with the calculation based on Cox’s experiment. The computed results show that the WINDWAVE model reasonably generated the bed shear stress by wave in comparison with the experimental data. At the same time, it also shows that the ratio of Nikuradse equivalent sand grain and bed roughness is very important for bed shear stress calculation, because this ratio together with the wave Reynolds number, decides the turbulent regime of the boundary layer.

Keywords: Wind-induced wave; Numerical simulation; EFDC model

1. INTRODUCTION

In general the influence of wind on the field of flow velocity is considerable for the problems of simulations of the hydrodynamic regime, sediment transport for lakes, estuarine and coastal areas with strong wind conditions. Wind effects can not only induce the flow current, but also generate surface waves with a wave height of up to several meters. Consequently, the calculation of the total bed shear stress must take the wave factor into account.

In fact, waves on the ocean surface with periods of 3 to 25 seconds are primarily caused by winds. Therefore, it can be seen that wind–induced waves are an important part of hydrodynamic models. The advantage of this Wind-wave sub-model is that it can be easily incorporated into the source code of a hydrodynamic model instead of running a separate wave model. This means that the changes in hydrodynamic parameters are immediately updated in the wave calculations. At the same time it does not take a lot of time for calculation as is the case with other wave models.

In this report the theoretical basis and tests of the wind wave sub-model that is incorporated into EFDC is presented in detail. The mathematical formulae are empirical equations called the SMB model (Sverdrup, Munk and Bretschneider, see Zhen-Gang Ji, 2008). The model calibration is based on the experiment of Cox et al. (Cox, 1996) for a wave flume. Another verification of the sub-model is the comparison of wave heights computed by the model with those generated by SWAN for the same wind condition in Caloosahatchee Estuary.

2. MATHEMATICAL FORMULAS IN WINDWAVE MODULE

2.1 Wave parameters generated by wind

The main wave parameters include wave height, wave direction and wave period. For the SMB model, the wave direction is the same as the wind direction. This means that the effects of refraction, diffraction and reflection are not taken into account. Wave height and period are as follows:

$$H = 0.283\alpha \frac{V_w^2}{g} tanh\left(\frac{0.0125}{\alpha}\left(\frac{gF}{V_w^2}\right)^{0.42}\right) \tag{1}$$

$$T = 7.54\beta \frac{V_w}{g} tanh\left(\frac{0.077}{\beta}\left(\frac{gF}{V_w^2}\right)^{0.25}\right) \tag{2}$$

$$\alpha = tanh\left(0.53\left(\frac{g\bar{h}}{V_w^2}\right)^{0.75}\right),\ \beta = \tanh\left(0.833\left(\frac{g\bar{h}}{V_w^2}\right)^{0.375}\right)$$

in which H is the wave height (m), T wave period (s), h the water depth (m), the whole domain average is noted by over bar, V_w velocity intensity of wind (m/s), and F the fetch of wind from the boundary to the cell in the wind direction (m) based on 16 directions. For its theoretical basis the SMB model used the following basic assumptions to derive Eqs. (1)-(2):

- The wind time for one direction is long enough to attain equilibrium
- The wind speed and water depth are spatially uniform over the fetch

In general the wave length, $L(m)$, is calculated from solving the non-linear equation for the dispersion relation. However, in this model it is calculated by using an approximate formula (Thanh, Phi Hung et al., 2008):

$$L = T\sqrt{\frac{gh}{\delta}} \tag{3}$$

$$\delta = \gamma + \frac{1}{(1 + 0.6522\gamma + 0.4622\gamma^2 + 0.0864\gamma^4 + 0.0675\gamma^5)}$$

$$\gamma = \left(\frac{2\pi}{\mathrm{T}}\right)^2 \frac{h}{g}$$

The regime of flow is determined through the wave Reynolds number, R_w and the relative bed roughness r:

$$R_w = \frac{UA}{\nu},\quad r = \frac{A}{k_s}, \tag{4}$$

in which A is the semi-orbital excursion, k_s the Nikuradse equivalent sand grain roughness and ω angle frequency.

It should be noted that the ripple geometry is strongly affected by the bed material characteristics (Chung and Van Rijn, 2003). The ripple steepness is a very important factor, because it decides the scale of bed roughness (Hitching and Lewis, 1999) and hence has a strong influence on vertical distribution of horizontal flow velocity.

The friction coefficient is calculated according to the regime of turbulent flow.

- Laminar flow:

$$f_w = 2R_w^{-0.5},\ R_w \leq 5.10^5 \tag{5}$$

- Smooth turbulent flow:

$$f_w = 0.09R_w^{-0.2},\ R_w > 5.10^5,\ r > 1.57 \tag{6}$$

- Rough turbulent flow:

$$f_w = Exp(5.2r^{-0.19} - 6),\ R_w > 5.10^5,\ r \leq 1.57 \tag{7}$$

$$f_{wmin} = 0.3$$

2.2 Radiation stress by wave and energy dissipation

In the case of waves, apart from the forces from currents, it is also necessary to add the forces from waves for the whole water column, such as radiation stresses or stresses due to the roller in breaking waves (Mengguo and Chongren, 2003; Robert J. Weaver,2004). However, in this model only radiation stress is taken into account. The radiation stress of a small amplitude wave is determined by the following simple formulas (Longuet-Higgins and Stewart, 1964):

$$S_{xx} = E\left(2n - \frac{1}{2} - nsin^2\theta\right),\ S_{xy} = S_{yx} = Ensin\theta cos\theta\ ,\ S_{yy} = E\left(2n - \frac{1}{2} + nsin^2\theta\right) \qquad (8)$$

where θ is wave direction with respect to x axis ($\theta = 0$ when wave direction is the same as the x axis), E wave energy (kg/s^2), k wave number, C_g wave group velocity and C wave celerity.

Wave energy per unit sea surface is determined by the formula for random wave and wave energy dissipation due to bottom friction and breaking wave is also taken into account.

3. TEST RESULTS AND CALIBRATION

3.1 Wind Wave on Flume

The experiment result on the Precision Wave Tank of the Ocean Engineering Laboratory at the University of Delaware (Cox, 2008) is used for the study. The size and shape of the wave flume are shown in **Error! Reference source not found.**. Total length of the tunnel is 33m; width is 0.6m; depth is 1.5m; bed slope is 1/35 and still water is 0.4m. The distances of L2-L6 from L1 are 0.24, 0.36, 0.48, 0.6 and 0.72 m respectively. The wave friction factor, f_w, was calculated based on a semi-theoretical expression by Jonsson, 1966 for fully rough turbulent flow:

$$\frac{1}{4\sqrt{f_w}} + log\frac{1}{4\sqrt{f_w}} = log\left(\frac{A}{k_s}\right) - 0.08 \qquad (9)$$

The calculation was implemented for four cases of the ratio between Nikuradse equivalent sand grain roughness and bed roughness at 6 locations as shown in Table 1.

Table 1 Relative roughness and wave friction factor

Line No./		Ks=30Z$_0$		Ks=15Z$_0$		Ks=60Z$_0$		Ks=2d50	
Method	Z$_0$	A/ks	fw	A/ks	fw	A/ks	fw	A/ks	fw
L1 A	0.0074	57.2	0.028	114.8	0.021	28.6	0.039	63.5	0.026
L2 A	0.0016	404.6	0.013	809.3	0.01	208.9	0.016	97.1	0.022
L3 A	0.0046	114.3	0.021	228.5	0.016	56.5	0.028	78.8	0.024
L4 A	0.0054	70.3	0.025	140.7	0.019	35.2	0.035	57	0.028
L5 A	0.0063	58.3	0.028	114.8	0.021	28.9	0.038	55.1	0.028
L6 A	0.0093	33.1	0.036	65.5	0.026	16.5	0.052	46.2	0.031

The bed shear stress is calculated according to Jonsson and Carlsen, 1976 as follows:

$$\tau_b(t) = \rho|u_*|u_*,\ u_* = \sqrt{(f_w/2)}U_b cos\omega t \qquad (10)$$

In addition, another formula was also used to calculate bed shear stress based on velocity measured just above the bottom boundary layer in the quadratic form (Grant and Madsen, 1979):

$$\tau_b(t) = 0.5\rho f_b |u_b| u_b \quad (11)$$

in which f_b is the bottom friction factor measured by using measured shear velocity and U_b near bottom maximum horizontal velocity given by Cox's experiment. The velocities in (10) and (11) are the time-average velocities for 50 waves (wave period is about 2.2s). Bed shear stress calculated with both the formulas (10) and (11) were used as the basis for the WINDWAVE sub-model comparison.

Based on four different cases of the ratio of Nikuradse roughness and bed roughness, k_s/z_0, from Cox's experiment (Table 1), four EFDC models corresponding to experimental cases were built to calculate bed shear stress. It should be noted that the waves and velocity field in EFDC for these cases were completely generated by wind. Therefore, an open boundary condition was not used here. The input parameters were selected so that the conditions of the numerical models were as similar to that of the experiment as possible.

The wind direction is along the flume. Wind speed is selected so that the computed wave heights are approximated with the values at the locations L1-L6 as shown in Figure 1. Two other parameters set up for EFDC included bed roughness and Nikuradse equivalent sand grain roughness. Based on the given ratio of k_s/z_0 and constant value of k_s in EFDC, an average bed roughness of 0.0058 m at six locations L1-L6 was used for the whole computed domain.

The input parameters for 4 tests (R1-R4) are presented in Table 2. Here the bed roughness, z_0, the ratio of Nikuradse roughness k_s, and bed roughness are important factors in evaluating the bed shear stress (Chung and Van Rijn, 2003; Hitching and Lewis, 1999). Then the values of coefficient k_s were calculated based on these given ratios and bed roughness values. For test R4 this ratio was not used and another method for determining k_s is introduced instead. That is, $k_s = 2d_{50}$ with $d_{50} = 1\,mm$. Two options of the WINDWAVE model: wave effects on the boundary layer only, and wave effects on whole water column, were carried out.

Table 2 The input parameters for 4 tests.

Tests	Bottom roughness (m)	Wind speed (m/s)	Wind direction (Deg.)	Ratio k_s/z_0	k_s (m)
R1	0.0058	29	90	30	0.1740
R2	0.0058	29	90	15	0.0870
R3	0.0058	29	90	60	0.3480
R4	0.0058	29	90	-	0.0025

Comparison between the model and experimental results at the six locations L1 to L6 for wave height and bed shear stress are shown in Figure 2-3. For Cox's experimental results, bed shear stress using wave friction factor, f_w, and bottom friction factor f_b are both shown. The RMS error based on computed results by WINDWAVE model and Cox's experiment is presented in Table 3.

Table 3 RMS Error for 4 tests in comparison with Cox's experiment

Test	ISWAVE=3			ISWAVE=4		
	(Ha)	(fw)	(fb)	(Ha)	(fw)	(fb)
R1	0.0251	0.2524	0.3593	0.0251	0.2522	0.3611
R2	0.0251	0.3654	0.3593	0.0251	0.3702	0.3611
R3	0.0251	0.3216	0.3593	0.0251	0.3137	0.3611
R4	0.0251	0.4049	0.3593	0.0251	0.4008	0.3611

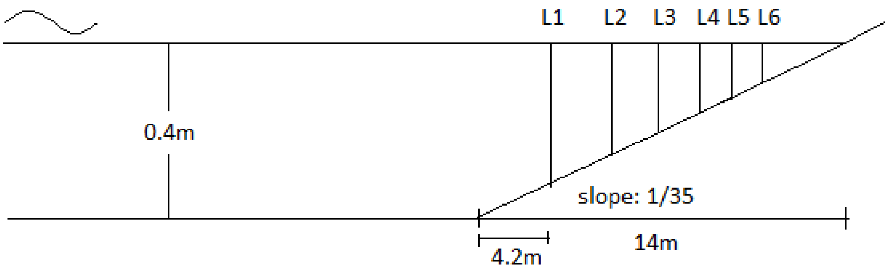

Figure 1 A sketch of the wave tank.

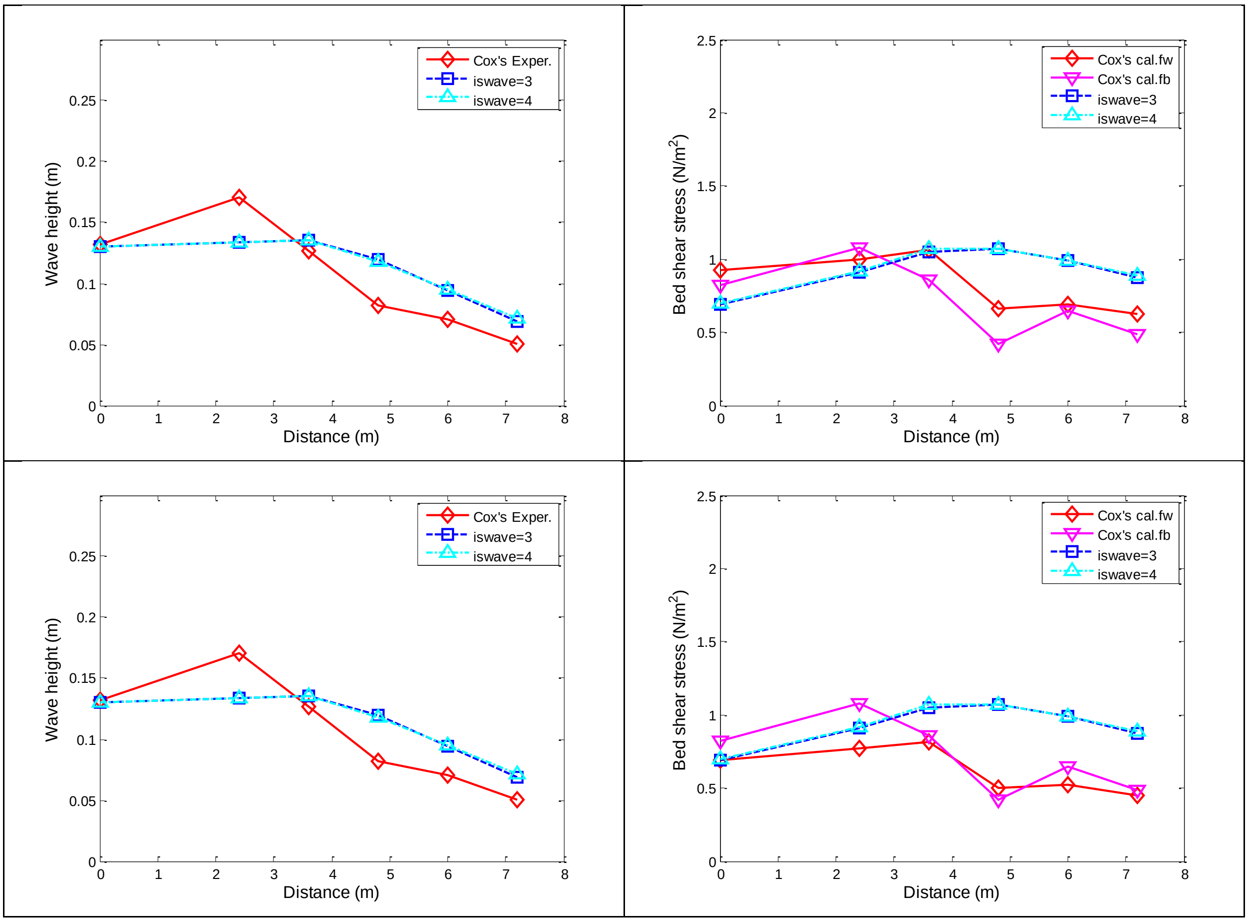

Figure 2 Comparison of windwave model and experiment at 6 locations L1-L6

Left: Wave heights *Right: Bed shear stress*
Top: Experiment R1 *Bottom: Experiment R2*

Columns 2 and 5 (Ha) are the RMS error for computed and measured wave heights; columns 3 and 6 (fw) are the RMS error for bed shear stress by (10) and WINDWAVE model; the columns 4 and 7 (fb) are the ones for bed shear stress by (11) and model. Figure 2-3 and Table 3 show that the wave heights generated by WINDWAVE for the four tests R1-R4 are the same, because wind-induced wave mainly depends on wind speed, direction and fetch. Bed shear stress by wave directly depends on relative bed roughness and the regime of turbulence via wave Reynolds number. The RMS errors of wave heights in the four tests R1-R4 for two options of wave calculation are quite large. This is due to the basic difference in the way to generate the wave in the flume, e.g. wave parameters and flow. Consequently, this difference certainly influences the bed shear stress calculated by experimental data and model. This was verified through the RMS evaluation in the columns (2)-(3) and (5)-(6). The main reason for the inaccuracy is most likely due to the differences in wave and flow conditions of the flume and model. Another reason is using different formulas. The bed shear stresses generated by two wave options are not significantly different, because the scale of flume is not large enough. Also from Table 3, the relation of $k_s = 30z_0$ should be used for more accuracy. In general, the hydrodynamic conditions set up for the model are not appreciably different from those in the wave flume and therefore the data from the experiment is quite suitable for calibrating the WINDWAVE model.

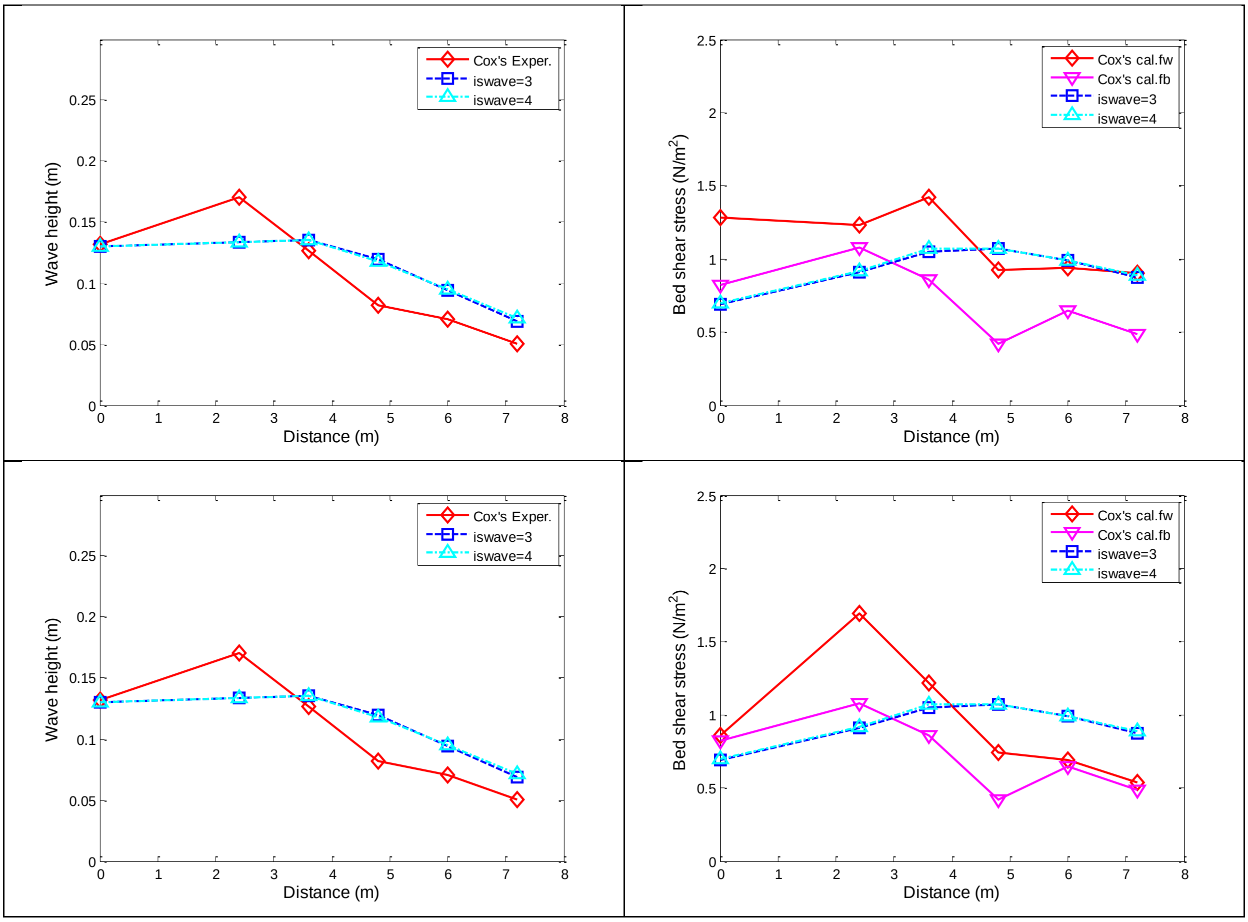

Figure 3. Comparison of windwave model and experiment at 6 locations L1-L6

Left: Wave heights *Right: Bed shear stress*
Top: Experiment R3 *Bottom: Experiment R4*

3.2 Wind wave application for Caloosahatchee

The computational domain to apply WINDWAVE is the Caloosahatchee Estuary which is covered by a curvilinear grid of 750 cells and the vertical direction is divided into 4 layers in the Sigma coordinates system. The options of wind wave for EFDC include:

- ISWAVE=2: Wave effect for the whole water column based on external wave generated by SWAN.
- ISWAVE=3: Wave is generated by WINDWAVE inside EFDC. In this case wave effect only influences the boundary layer.
- ISWAVE=4: Wave is also generated by WINDWAVE, but wave effect for the whole water column is taken into account in a way similar to that of ISWAVE=2.

In order to provide further comparisons on wave calculation by WINDWAVE with the other computational methods, WINDWAVE in the EFDC and SWAN models using the same wind condition are applied and compared. The process of wave effect calculation on turbulence for Caloosahatchee includes the following steps:

- Set up and run EFDC for Caloosahatchee Hydrodynamics for 2 days to generate water level and velocity field for SWAN run.
- Run SWAN with water level and velocity field from EFDC to obtain wave parameters: wave height, direction, period and radiation stress components for the next run of EFDC with the case ISWAVE=2.
- Set up and run EFDC with the case ISWAVE=3 using WINDWAVE sub-model.
- Set up and run EFDC with the case ISWAVE=4 using WINDWAVE sub-model.

A constant wind field of 15 m/s from SSW direction was used for these cases. The duration of simulation is 2 days.

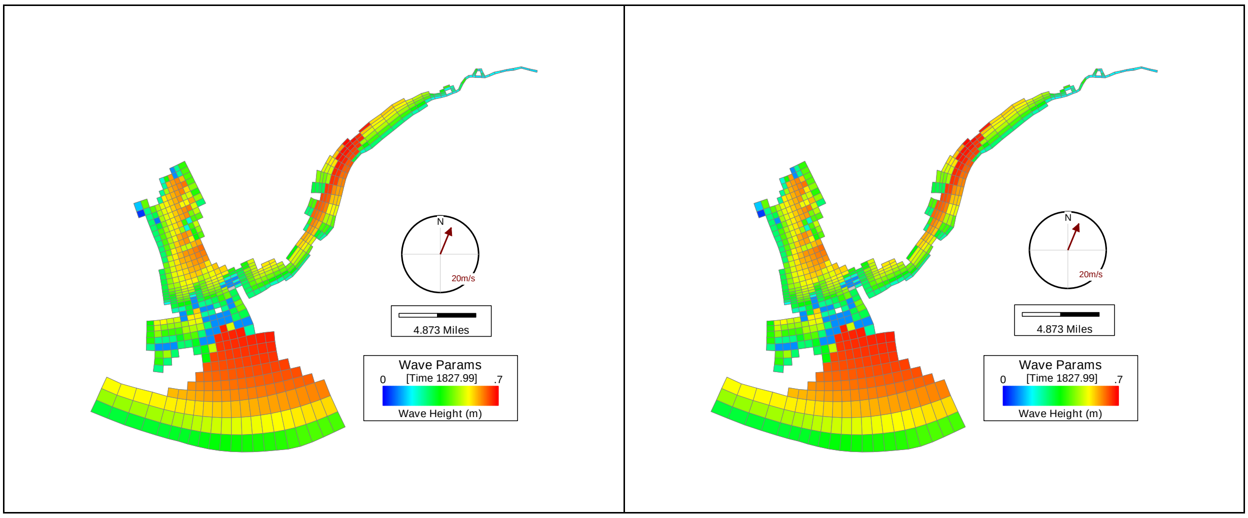

Figure 4. Left: Wave height calculated by EFDC after 2 days (ISWAVE=3)
Right : Wave height calculated by EFDC after 2 days (ISWAVE=4)

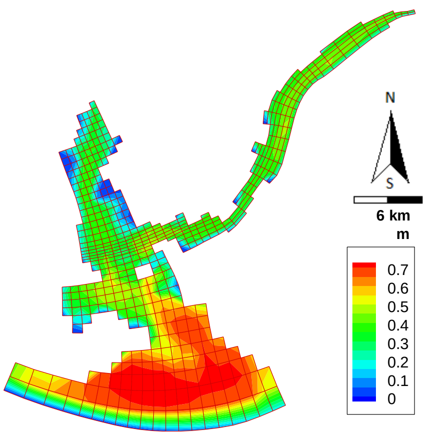

Figure 5. Wave height by SWAN with the same wind condition

From these computed results it shows that wave heights and bed shear stresses in cases ISWAVE=3 and ISWAVE=4 (Figure 4) are slightly higher than those of ISWAVE=2 (Figure 5). This is due to the phenomena of refraction and diffraction were not taken into account in WINDWAVE. In general, the behavior and height of waves by WINDWAVE (Figure 4) and by SWAN (Figure 5) are quite similar except at the bend of the branch of the river. The difference here is due to refraction and diffraction, which is ignored in the WINDWAVE module.

4. CONCLUSIONS

The WINDWAVE model developed in EFDC was calibrated through the comparison with the calculation based on Cox's experiment. The computed results show that the WINDWAVE model reasonably generated the bed shear stress by wave in comparison with the experiment data. At the same time, it also shows that the ratio of Nikuradse equivalent sand grain and bed roughness is very important for bed shear stress calculation, because this ratio together with the wave Reynolds number, decides the turbulent regime of the boundary layer. The formula $k_s = 30z_0$ is recommended to use for bed shear stress calculation. Generating internal wave based on the wind field makes EFDC very convenient for simulating the problems relating to the hydrodynamic regime, sediment transport, etc. in conditions of strong winds where the factor of bed shear stress by wave cannot be ignored. In addition, with the internal wave option the wave field will be calculated for every time step, while this feature is restricted for the external wave.

5. ACKNOWLEGEMENTS

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6. REFERENCES

Chung D H and Van Rijn L C (2003). Diffusion approach for suspended sand transport under waves, J. *Coast. Res.* **19** 1–11, pp.1-11.

Cox, Daniel T., Nobuhisa Kobayashi and Akio Okayasu (1996). Bottom shear stress in the surf zone, *J. of Geophysical Research*, Vol. 101, N.C6, pp.14337-48.

Grant W D and O S Madsen (1979). Combined wave and current interaction with a rough bottom, *J. Geophys. Res*., **84**(C4), 1797-1808

Hitching,E. and Lewis, A.W. (1999). Bed roughness over vortex ripples. *Proceedings of the 4th International Symposium on Coastal Engineering and Coastal Sediment Processes*, Long Island, New York ASCE, pp.18-30.

Jonsson,I.G (1966). Wave boundary layers and friction factors, in *Proceedings of the 10th Coastal Engineering Conference*, pp. 127-148, Am. Soc. of Civ. Eng., New York, 1966.

Jonsson, I G and N A Carlsen (1976). Experimental and theoretical investigations in an oscillatory turbulent layer, *J. Hydraul. Res*., **14**,45-60.

Longuet-Higgins, M.S., Stewart, R.W. (1964). Radiation stress in water waves, a physical discussion with application. *Deep-Sea Res*. 11, 529– 563.

Mengguo,LI, Chongren QIN (2003). Numerical simulation of wave-induced nearshore current, *International Conference on Estuaries and Coasts Nov* 9-11,2003.

Robert J. Weaver (2004). Effect of wave forces on storm surge, *Master of Science, University of Florida*

Soulsby R. (1997). Dynamics of Marine Sands, *London: Thomas Telford Publications.*

Thanh, Phi Hung X., Matthew D. Grace, and Scott C. James (2008). *Sandia National Laboratories Environmental Fluid Dynamics Code: Sediment Transport User Manual.*

Zhen-Gang Ji (2008). Hydrodynamics and Water Quality modeling Rivers, Lakes and Estuaries. *John Wiley & Sons, Inc.*